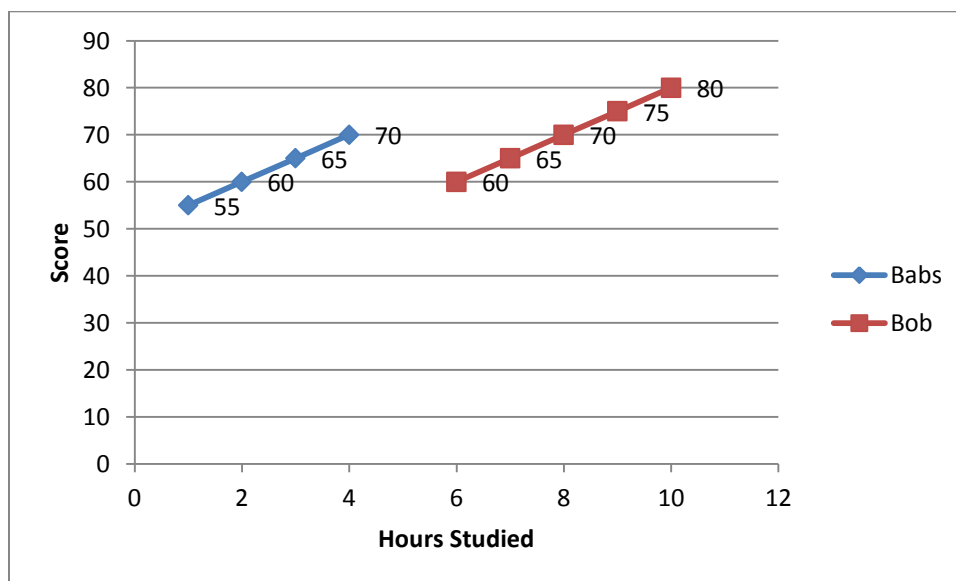# Gregarious Consulting

Lifelong Learning, Lifelong Tested

Calculus Diagnostic Solutions

Answer the following questions as briefly as possible. Show work whenever possible for partial credit. Unless otherwise stated, the time limit is 60 minutes. You may use the same resources here as you would use in your real assessment (course exam, employment exam, etc.).
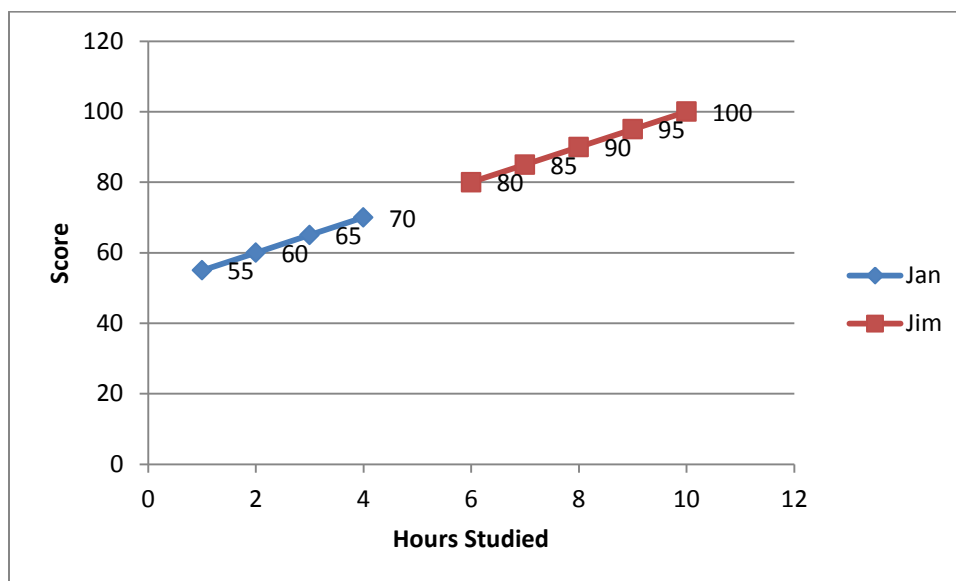
1) Bob and Babs are studying for calculus. Bob used to study for 10 hours total per week, but now he is studying less, and his scores are slipping. Babs used to study for 1 hour total per week, but now she is studying more, and her scores are rising.

a) If Bob and Babs both end up studying for 5 hours each per week, do you think they will get the same score? *No. Stretch the lines, and they will not connect at the same score.*

b) If so, what do you expect that score to be?

c) For every 1 additional hour per week Babs studies (e.g. switching from 2 hours to 3 hours), Babs gains <u>5</u> points on her score.

d) Predict Babs' score if she does not study at all: *0*

e) Predict Babs' score if she studies for 6.5 hours: *82.5*

f) Write a function or relationship to predict Babs' score for any time investment:

   *Bab's Score = 50 + 5 * Hours Studied*

2) Jim and Jan are also studying for calculus. Jim is naturally better at calculus than Bob, but Jim also shares Bob's bad decision of studying less and slipping. Jan started studying only 1 hour per week, but now she is studying more, and her scores are rising.

a) If Jim and Jan both end up studying for 5 hours each per week, do you think they will get the same score? *Yes*

b) If so, what do you expect that score to be? *75*

3) Y = 10 + 2X

Fill in the table:

X	Y
-1	8
0	10
1	12
2	14

4) f(X) = 20 - 3X

Fill in this table also:

X	f(X)
-1	23
0	20
1	17
2	14

5) Greg drove for 3 hours. He traveled about 180 miles. On average, how fast did he drive? Show your work.
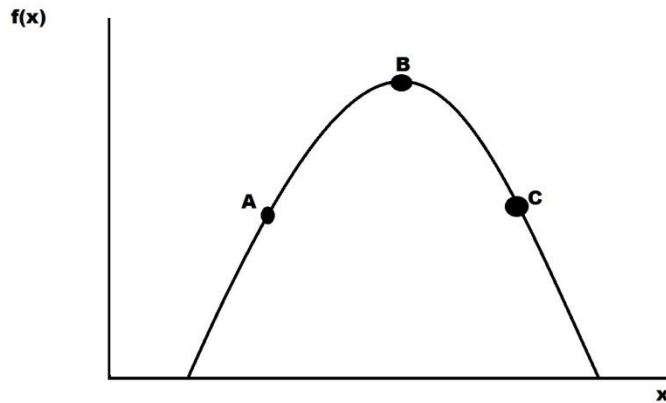
$$\frac{180\ miles}{3\ hours} = \frac{60\ miles}{1\ hour} = 60mph$$

6) $Slope = \frac{Rise}{Run} = \frac{\Delta Dependent\ Variable}{\Delta Independent\ Variable} = \frac{\Delta Y}{\Delta X} = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{\Delta Output}{\Delta Input} = Rate\ of\ Change$

a) What was the average speed when traveling from t =1 to t =5? That is, in those 4 hours, what was the average speed? $\frac{(20-12)miles}{(5-1)hours} = 2mph$

b) What was the average speed when traveling from t = 2 to t=4? *2mph*

c) What was the average speed when traveling form t = 2.9 to t = 3.1? *2mph*

d) Suppose you took a snapshot at t = 3 (technically, 0 hours have passed in that snapshot). According to algebra, what was the average speed? $\frac{(16-16)}{(3-3)} = \frac{0}{0}$ indeterminate

e) Suppose you do not believe in algebra now. If you were observing this person in motion from start to finish and took a snapshot at t=3, what do you think the speed really was at the time? *2 mph*

Questions 7 to 12 are of importance to folks in finance, operations, and marketing.
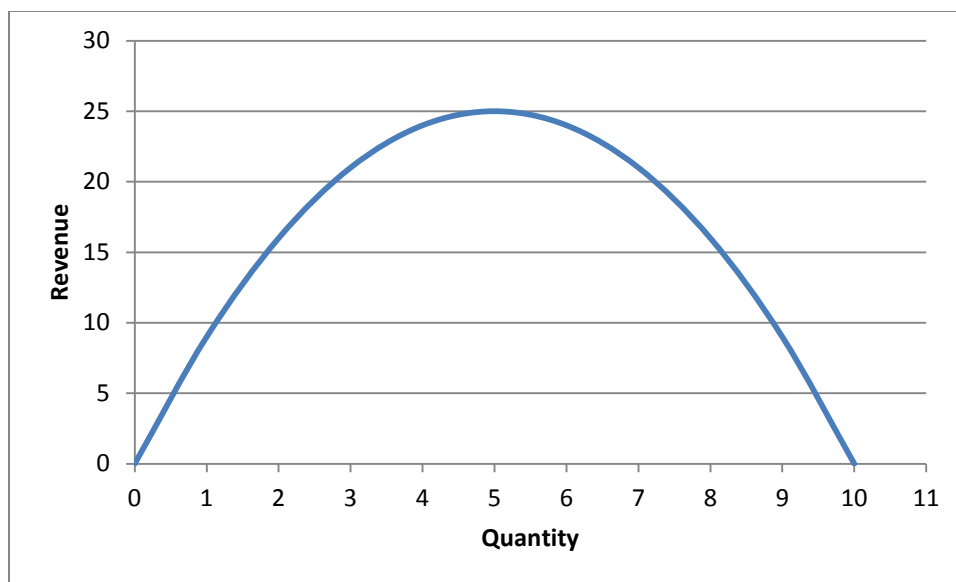
7)

Let slope $= \dfrac{\Delta f(x)}{\Delta x} = \dfrac{rise}{run}$

a) At point A, the slope is:
   - *a. Positive*
   - b. Negative
   - c. Zero, or Flat

b) At point B, the slope is:
   - a. Positive
   - b. Negative
   - *c. Zero, or Flat*

c) At point C, the slope is:
   - a. Positive
   - *b. Negative*
   - c. Zero, or Flat

d) Which of those 3 is the maximum point on the graph? *B*

e) What can we infer about the maximum point and its slope?

The slope of the maximum point could be 0.

8) You are selling tickets to a small play for children.

    a) There are 10 seats available. An advisor suggests that you sell out (i.e. sell all 10 tickets). If your incentive is to maximize revenue, do you agree with the suggestion?

No, because if 10 tickets are sold, the revenue will be 0.

    b) If you disagree, how many tickets would you sell? *5 tickets.*

9) Revenue = Price * Quantity. For example, if you sold 2 apples, and the price of each apple were $0.29, then you earned $0.58 revenue.
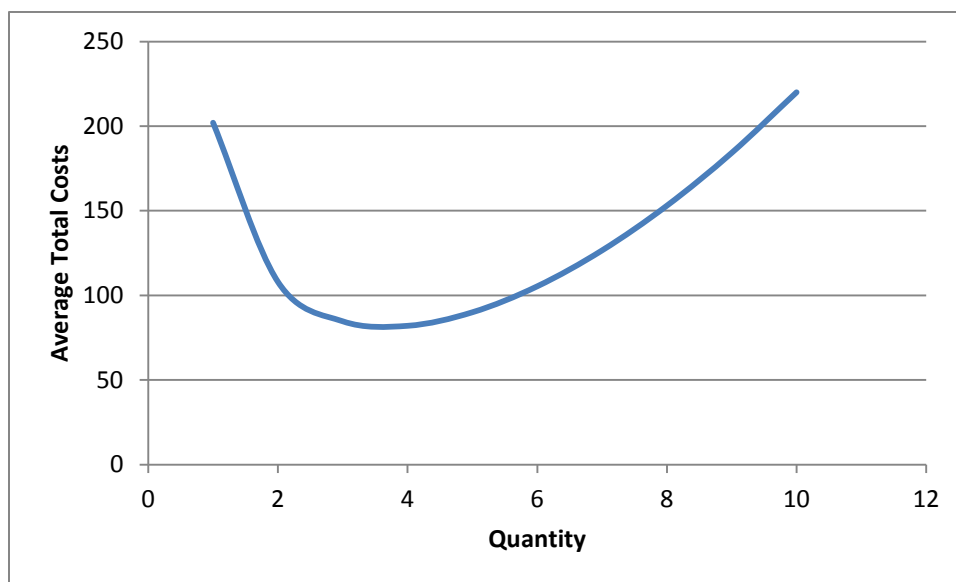
Let Price = 6 – Quantity

    a) Fill in the Table:

Quantity	0	1	2	3	4	5	6
Price	6	5	4	3	2	1	0
Revenue	0	5	8	9	8	5	0

    b) What price and quantity yield the maximum revenue? $3 and 3 units.
    c) If you graphed a curve with Quantity as the independent variable, and Revenue as the dependent variable, what would be true of the slope of the curve from your answer to (b)?

The slope would be 0 because the revenue is maximum.

10) The following graph shows a relationship between quantity and unit costs.

a) According to the graph, how many units would you produce to minimize unit costs? *4*
b) What do you notice about the slope given your answer to (a)? *Slope = 0*
c) Complete this sentence: "When we want to find the maximum or minimum of a function, whether for revenue or costs or profits or anything, we first seek where *slope = 0*."

11) *Let y = $3x^2$*

So y is the output, or dependent, variable.

And x is the input, or independent, variable.

The coefficient is the number in front of the variable. What is the value of the coefficient? 3

The power is the number to the upper right of the variable. What is the value of the power? 2

12) Magic Trick:
   a) Multiply the power by the coefficient above. Record that number here: 6
   b) Subtract the power by 1. Record that number here: 1
   c) Let the answer from (a) be the new coefficient for x. Let the answer from (b) be the new power for x. What did 3x^2 turn into? 6x
   d) Let y = 10x^3 and use the magic trick again. What did you get?

(3)(10)x^(3-1) = 30x^2
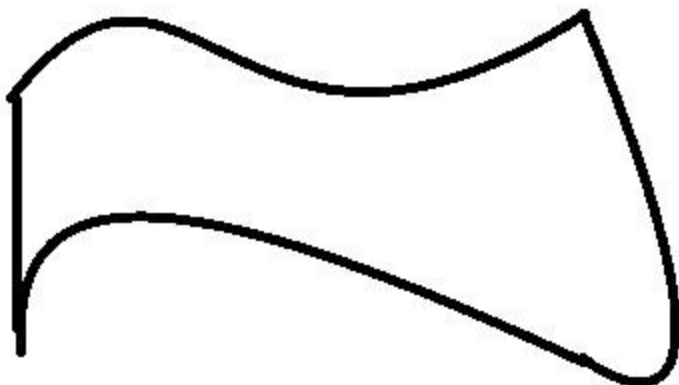
Questions 13 to 16 are of importance to engineers.

13) The area of a square is the square of its side length. The length of a square is 7. What is its area? 7^2 = 49

14)
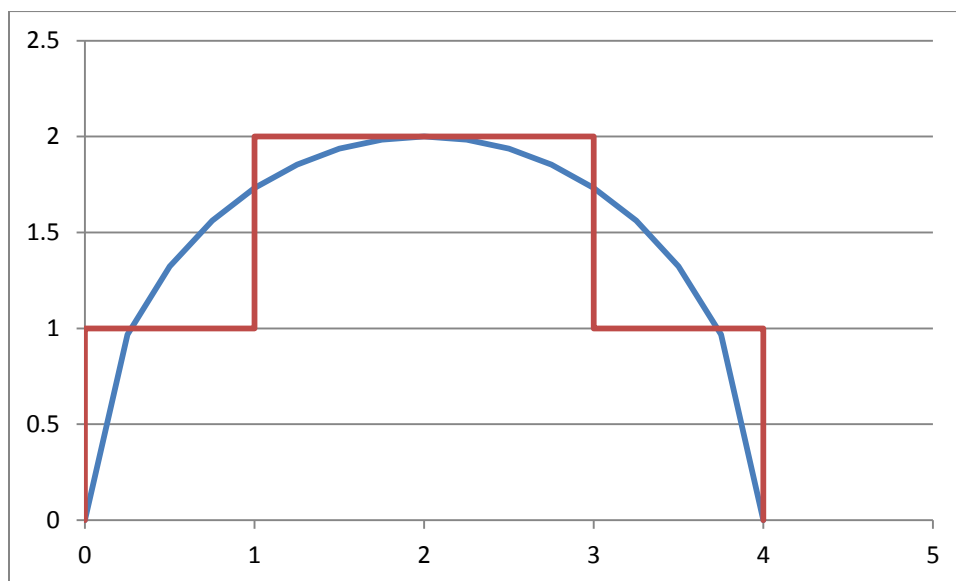

Above is a square with length x. What is its area? X^2

15)

How would you get the area of the "Frito® chip"[1] above?

a) I bet there's a magic formula that can solve anything
b) It's mathematically impossible because it is too irregular
c) I don't know how to get the exact area, but if I stacked a bunch of tiny rectangles to fit closely to that shape and added their areas together, I could approximate the answer.

16)

Using rectangles of any length or width of your choice, estimate the area between the curve and the horizontal axis. *Answers may vary. Here, rectangles have a length of 1 outlined in red. Total area of rectangles = 1 + 2 + 2 + 1 = 6. This semi-circle's actual area is about 6.28.*

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[1] Frito® is a registered trademark of Frito-Lay, Inc.

Scoring

Visual (Red)

Total your scores for: 1a, 1d, 2a, 6c, 7a, 7b, 7c, 7d, 8a, 8b, 10a

If fewer than 8, you might need to incorporate tracing habits with graphs.

Numerical Patterns (Blue)

Total your scores for: 1c, 1e, 2b, 6b, 9b

If fewer than 4, you might need to iteratively add various numbers.

Symbolic (Purple)

Total your scores for: 1f, 11, 12c, 14

If fewer than 3, you might need to write a verbal relationship in a sentence before writing the algebraic equation.

Computation (Green)

Total your scores for: 3, 4, 6a, 6d, 9a, 12a, 12b, 13

If fewer than 6, you might need to rewrite the formula and then fill in all the numbers, or you might need a calculator.

Intuition/Processing/Application (Brown)

Total your scores for: 5, 6e, 7e, 9c, 10b, 10c, 12d, 15, 16

If fewer than 7, you might need to observe more real-life phenomena and temporarily set aside thinking of numbers when facing these questions.